A conjecture linking graph theory and efficient simulation of measurement-based quantum computation

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Quantum computational power

If quantum computers are more powerful than classical computers, exactly what features make them more powerful?

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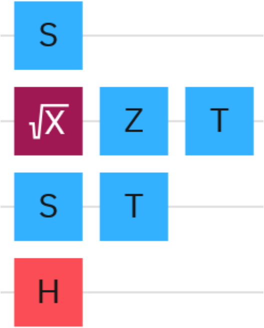
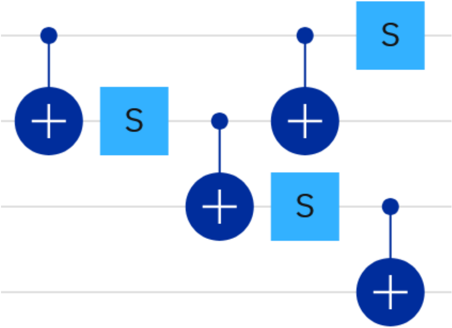
Related question: **when do quantum computers have the same computational power as classical computers?**

Quantum circuits with only local gates



No entanglement → The quantum computation can be efficiently simulated on a classical computer.

Quantum circuits with only Clifford gates



Theorem (Gottesman-Knill, 1998)

A quantum computation consisting only of Clifford gates can be efficiently simulated on a classical computer.

Sources of computational power

Two potential sources of quantum computational power:

- 1) Entanglement;
- 2) non-Clifford operations.

Measurement-based quantum computation

The idea behind **measurement-based quantum computation** is to separate the two potential sources of quantum computational power.

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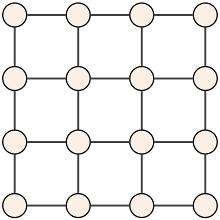
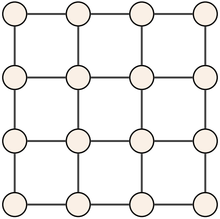
- 1) Prepare a **graph state** with only CZ gates (Clifford gates);



Measurement-based quantum computation

The idea behind **measurement-based quantum computation** is to separate the two potential sources of quantum computational power.

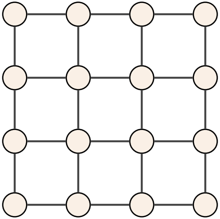
- 1) Prepare a **graph state** with only CZ gates (Clifford gates);
- 2) **Measure** qubits in some ways that may depend on the previous measurements (no entanglement added).



Universality of measurement-based quantum computation

Theorem (Briegel, Raussendorf, 2001)

Measurement-based quantum computation on the grid graph states has the same computational power as quantum circuits.



We say that the grid graph states (also called the 2D cluster states) are **universal resources**.

Reformulation of the question

In measurement-based quantum computation, the question

"**What features make quantum computers powerful?**"

becomes

"**Which graph states are universal resources?**"

Reformulation of the question

In measurement-based quantum computation, the question

"**What features make quantum computers powerful?**"

becomes

"**Which graph states are universal resources?**"

Alternatively:

"**For which graph states is measurement-based quantum computation efficiently classically simulable?**"

The simulation conjecture of Geelen

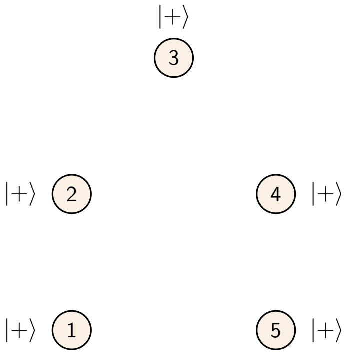
Conjecture (The simulation conjecture)

If we restrict ourselves to preparing **graph states** *from any* **proper vertex-minor-closed class** *of graphs, then measurement-based quantum computation is* **efficiently classically simulable***.*

Graph states and vertex-minors

Graph states

A graph state is a quantum state represented by a graph. The vertices represent the qubits and the edges represent entanglement.



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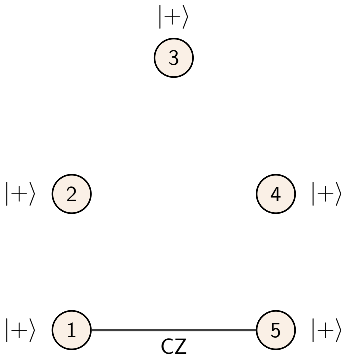

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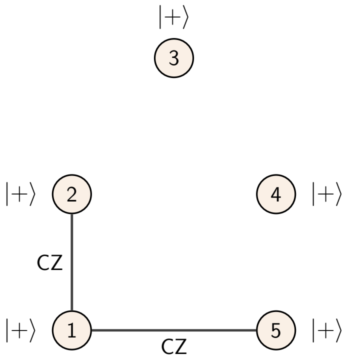

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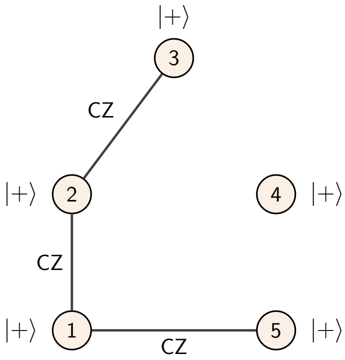

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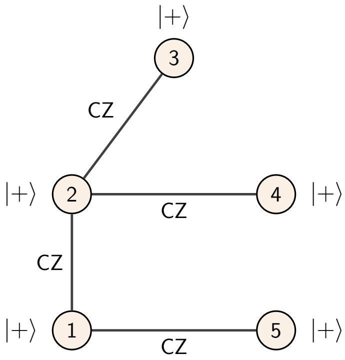

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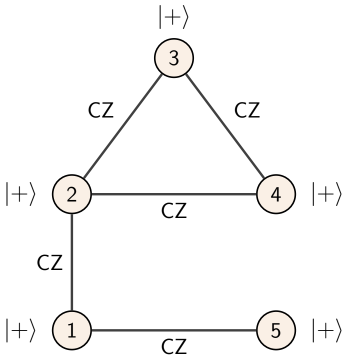

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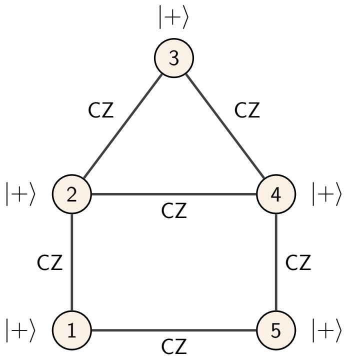
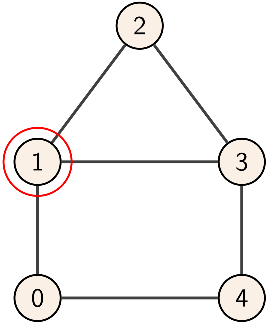


Local complementation

Definition

A local complementation on a vertex u consists in complementing the (open) neighborhood of u.



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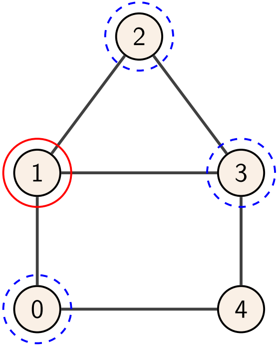

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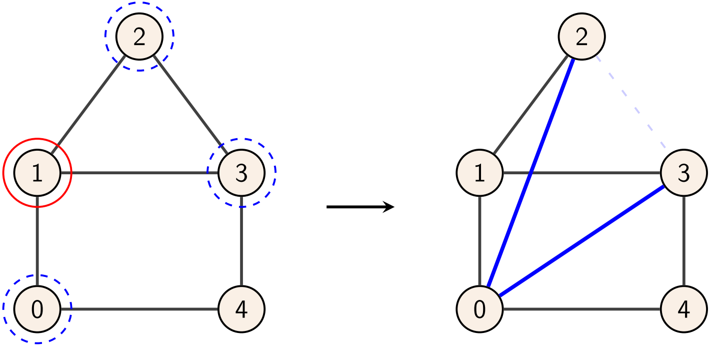

Vertex deletion

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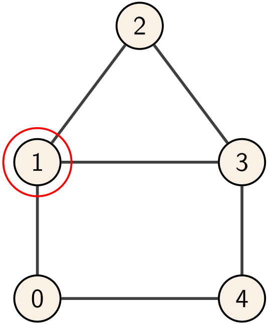
A vertex deletion on a vertex u consists removing u and its adjacent edges from the graph.



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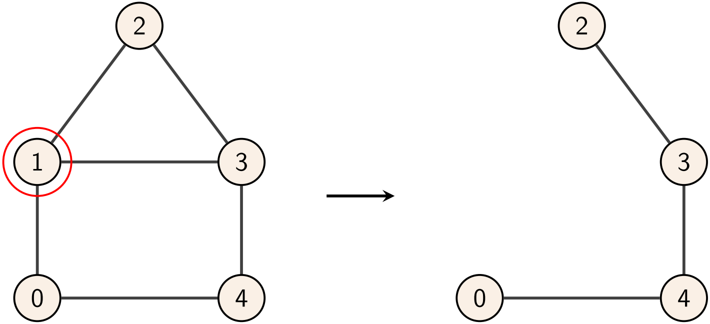
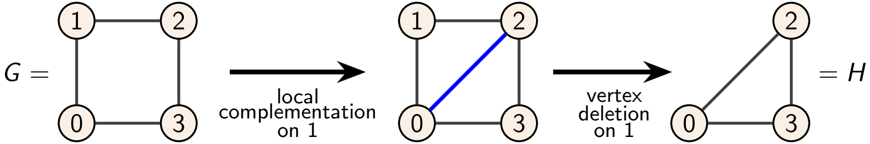


Vertex-minors

Definition (Vertex-minor)

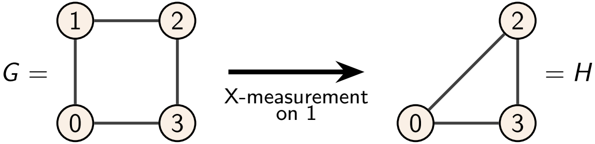
H is a vertex-minor of G if H can be obtained from G by means of local complementations + vertex deletions.



Vertex-minor = local Clifford

Theorem (Dahlberg, Wehner, 2018)

If H is a vertex-minor of G then $|H\rangle$ can be obtained from $|G\rangle$ by local Clifford gates, local Pauli measurements, and classical communication.



Vertex-minor closed class of graphs

Definition

A class of graph $\mathcal{G}$ is **vertex-minor closed** if any vertex-minor of some $G \in \mathcal{G}$ is also in $\mathcal{G}$.

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Any vertex-minor closed class of graphs can be characterized by a **finite** *set of forbidden vertex-minors.*

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Definition

A vertex-minor closed class of graphs is **proper** if it does not contain every graph.

The simulation conjecture of Geelen

Conjecture

If we restrict ourselves to preparing **graph states** *from any* **proper vertex-minor-closed class** *of graphs, then measurement-based quantum computation is* **efficiently classically simulable***.*

Reformulation:

Conjecture

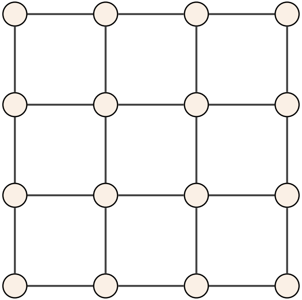
If we restrict ourselves to preparing **graph states** *from any* **class of graphs that is closed under local complementation and vertex deletion, but not composed of all graphs***, then measurement-based quantum computation is* **efficiently classically simulable***.*

Outline

- A counter-example
- Example 1: graphs of rank-width at most k
- Example 2: Circle graphs
- Is the simulation conjecture well-formulated?
- Towards proving the simulation conjecture

A counter-example

The square grid



The class of grids is **universal** for measurement-based quantum computation.

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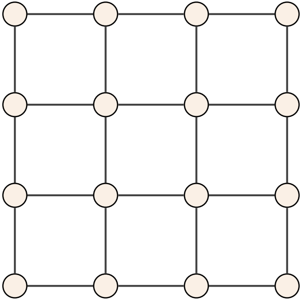

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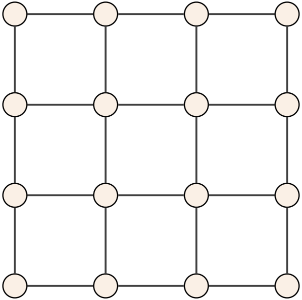

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- The class of 2D square grids is not vertex-minor closed.

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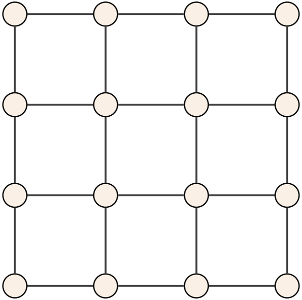

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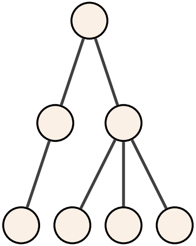
- The class of 2D square grids is not vertex-minor closed.
- The vertex-minor closure is not proper, i.e. it is the class of all graphs.

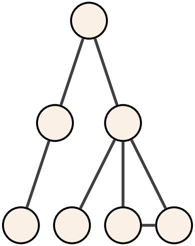
Rank-width

Tree-width

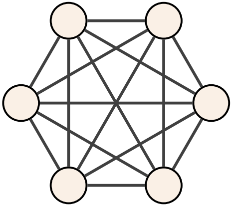
Informally, tree-width is a function in $\mathbb{N}$ that measures how close a graph is to a tree.



tree-width $= 1$



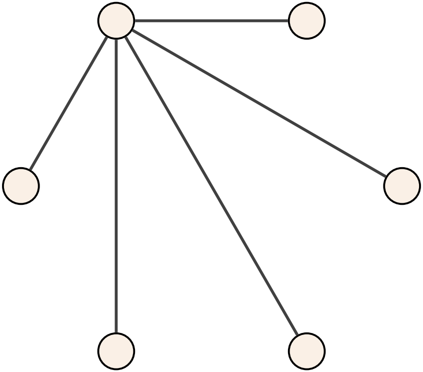
tree-width $= 2$



complete graph
on n vertices:
tree-width $= n - 1$

Tree-width does not pair well with local complementation

A star and a complete graph are the same up to local complementation.



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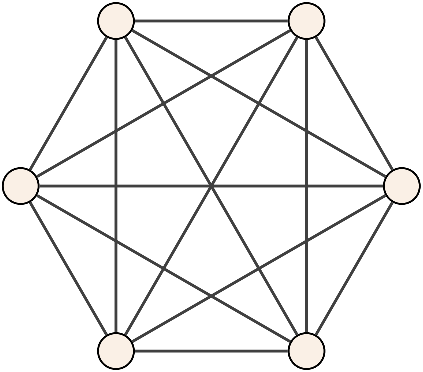
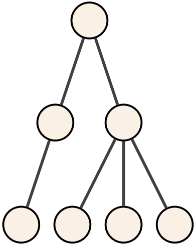


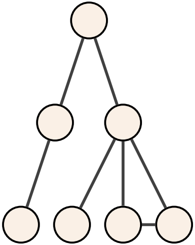
tree-width $= n - 1$

Rank-width

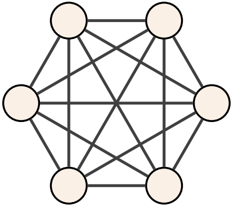
Informally, rank-width is like tree-width, but invariant by local complementation.



rank-width = 1



rank-width = 2



complete graph
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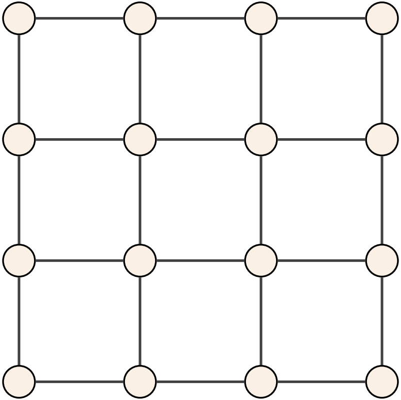
Rank-width of the square grid

The $m \times m$ square grid has rank-width $m - 1$ (Jelínek, 2008).



Low rank-width implies efficient classical simulation

Theorem (Van den Nest et al., 2006)

If the **rank-width** *of the graphs in a class $\mathcal{G}$ grows at most* **logarithmically** *with the number of qubits, then measurement-based quantum computation on $\mathcal{G}$ is efficiently classically simulable.*

In particular, measurement-based quantum computation is efficiently classically simulable if the rank-with is bounded.

A vertex-minor closed class of graphs

Rank-width is non-increasing under the vertex-minor relation. $\Longrightarrow$

Proposition

For a fixed integer k, graphs of rank-width at most k form a proper vertex-minor closed class of graphs.

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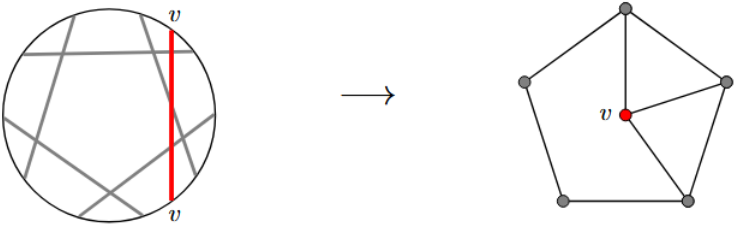
This is coherent with the simulation conjecture.

Circle graphs

Circle graphs: definition

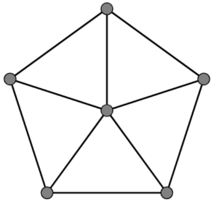
Definition

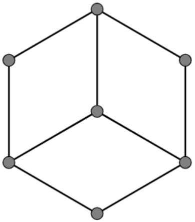
A circle graph is the intersection graph of a chord diagram.

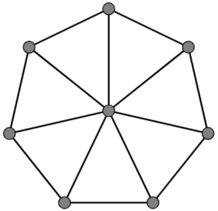


Forbidden minors of circle graphs

Alternative definition: circle graphs are those graphs which do not have one of the 3 graphs below as a vertex-minor (Bouchet, 1994).







Importance of circle graphs

Circle graphs play in the theory of **vertex-minors** the role that **planar graphs**[1] play in the theory of **minors**[2].

[1] planar = can be drawn in the plane without edges crossing.
[2] minor = obtained by vertex/edge deletion and edge contraction.

Importance of circle graphs

Circle graphs play in the theory of **vertex-minors** the role that **planar graphs**[1] play in the theory of **minors**[2].

Theorem (Grid theorem, Robertson and Seymour, 1986)

For any **planar graph** *G, there exists an integer r_G such that every graph with* **tree-width** *at least r_G has G as a* **minor***.*

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Theorem (Grid theorem for vertex-minors, Geelen et al., 2020)

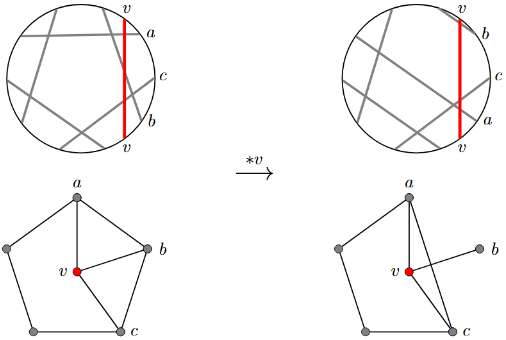
For any **circle graph** G*, there exists an integer* r_G *such that every graph with* **rank-width** *at least* r_G *has* G *as a* **vertex-minor***.*

[1] planar = can be drawn in the plane without edges crossing.

[2] minor = obtained by vertex/edge deletion and edge contraction.

Vertex-minors of circle graphs

Vertex deletion: removing a chord from the chord diagram.
Local complementation: "flipping" some chords.



Proposition

Circle graphs are a proper vertex-minor closed class of graphs.

Simulation of circle graphs

Very recent result:

Theorem (Harrison et al., 2025)

Measurement-based quantum computation is efficiently classically simulable on **circle graphs**.

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This is the first example of a class of graphs whose rank-width grows polynomially with the number of vertices, but on which measurement-based quantum computation is efficiently classically simulable.

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This result is coherent with the simulation conjecture.

Is the conjecture well-formulated?

The simulation conjecture of Geelen

Conjecture (The simulation conjecture)

If we restrict ourselves to preparing graph states from any proper **vertex-minor***-closed class of graphs, then measurement-based quantum computation is efficiently classically simulable.*

Limits of the vertex-minor formalism

Problem: the vertex-minor formalism only describes transformations of graph states that only use **local Clifford gates, local Pauli measurement, and classical communication**.

Limits of the vertex-minor formalism

Problem: the vertex-minor formalism only describes transformations of graph states that only use **local Clifford gates, local Pauli measurement, and classical communication**.

However, in the context of measurement-based quantum computation, all operations in LOCC (e.g. local unitaries not in Clifford) are allowed.

Local equivalences of graph states

Definition

Two quantum states are local unitary (LU) -equivalent if they are related by single-qubit unitary gates.

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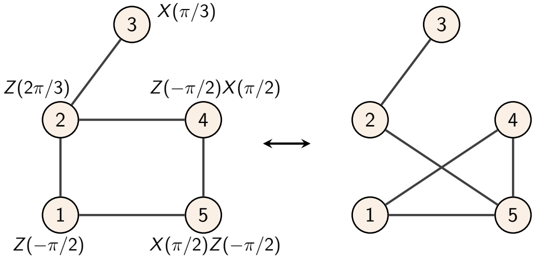
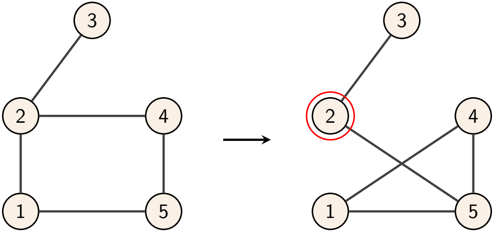
Two quantum states are local Clifford (LC) -equivalent if they are related by single-qubit Clifford gates.



Local complementation captures LC-equivalence

Theorem (Van den Nest, Dehaene, De Moor, 2004)

Two graph states are LC-equivalent iff the two corresponding graphs are related by **local complementations**.



LU $\neq$ LC

Theorem (Ji et al., 2008)

There exist graph states that are LU-equivalent but not LC-equivalent.

LU $\neq$ LC

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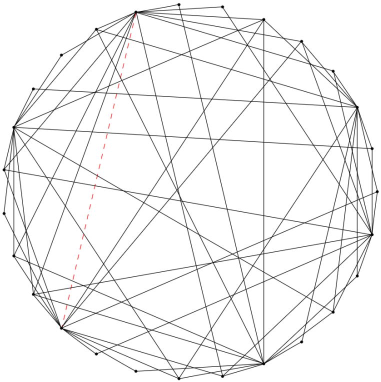
There exist graph states that are LU-equivalent but not LC-equivalent.

$\rightarrow$ A 27-qubit counterexample to the LU=LC conjecture.



r-local complementation

r-local complementation is a generalization of local complementation that captures LU-equivalence:

Theorem (C, Perdrix, 2025)

Two graph states are LU-equivalent iff the two corresponding graphs are related by r-local complementations for some r.

Ongoing work: LU-equivalence of circle graphs

Theorem (Harrison et al., 2025)

Measurement-based quantum computation is efficiently classically simulable on any state LU-equivalent to ***circle graphs****.*

Ongoing work: LU-equivalence of circle graphs

Theorem (Harrison et al., 2025)

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Ongoing work:

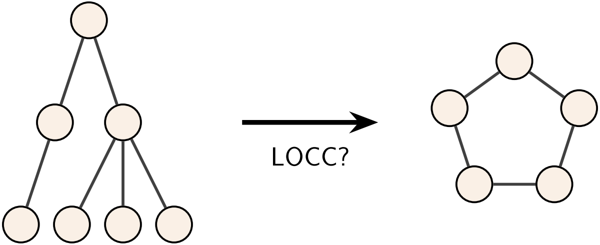
Proposition

Circle graphs are in fact closed by LU-equivalence, i.e. a graph state LU-equivalent to a circle graph state must be a circle graph state itself.

The proof makes use of r-local complementation.

Capturing LOCC transformations between graph states

Problem: how to tell graphically if a graph state $|H\rangle$ can be obtained from a (possibly bigger) graph state $|G\rangle$ by LOCC ?



A necessary condition

You can tell that $|H\rangle$ can **not** be obtained from $|G\rangle$ by LOCC if some entanglement measure is higher for $|H\rangle$ than for $|G\rangle$.

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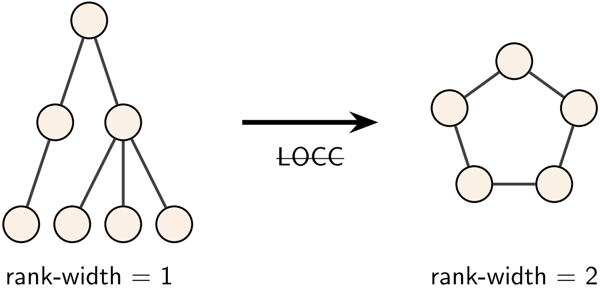
You can tell that $|H\rangle$ can **not** be obtained from $|G\rangle$ by LOCC if some entanglement measure is higher for $|H\rangle$ than for $|G\rangle$.

Proposition

If the rank-width of H is strictly higher than the rank-width of G, then $|H\rangle$ can **not** *be obtained from $|G\rangle$ by LOCC.*



Capturing LOCC (ongoing work)

Question: Does r-local complementation + vertex-deletion capture LOCC transformations between graph states?

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If it is the case → notion of r-vertex minor, and maybe a better formulation of the simulation conjecture is:

Conjecture

If we restrict ourselves to preparing ***graph states*** *from any* ***proper r-vertex-minor-closed class*** *of graphs, then measurement-based quantum computation is efficiently classically simulable.*

A direction towards proving the simulation conjecture

Towards proving the simulation conjecture

Conjecture (Informal statement, Geelen)

For any proper vertex-minor-closed class of graphs $\mathcal{G}$, each graph in $\mathcal{G}$ "decomposes" into parts that are "almost" circle graphs.

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Step 1: Proving this conjecture.

Step 2: Proving that measurement-based quantum computation on "almost" circle graph states is efficiently classically simulable.

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Step 1: Proving this conjecture.

Step 2: Proving that measurement-based quantum computation on "almost" circle graph states is efficiently classically simulable.

Step 3: Proving that measurement-based quantum computation is efficiently classically simulable on a graph state, if it is the case from every graph state in its "decomposition".

Thanks