Small k-pairable states

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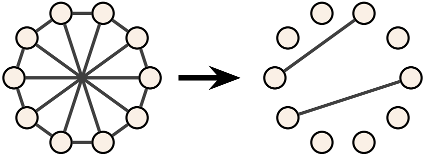


Outline

1. Graph states and k-pairability
2. A graphical interpretation of k-pairability using vertex-minors
3. Main result : Existence of small k-pairable states
4. k-vertex-minor universal graphs
5. Conclusion

Graph states and k-pairability

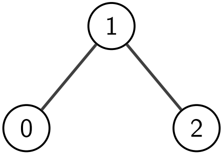
Graph states

Definition (Graph state)

Given a graph $G = (V, E)$, a graph state $|G\rangle$ is a quantum state written as :

$$|G\rangle = \left(\prod_{(u,v)\in E} CZ_{u,v} \right) |+\rangle_V$$

where $CZ_{u,v}$ is the controlled-Z gate acting on the qubits u and v.



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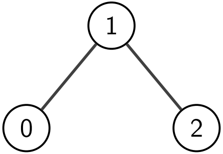
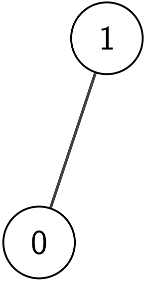


$$\begin{aligned} |G\rangle &= CZ_{0,1} CZ_{1,2} \left(|+\rangle_0 \otimes |+\rangle_1 \otimes |+\rangle_2 \right) \\ &= \frac{1}{\sqrt{8}} \left(|000\rangle + |001\rangle + |010\rangle - |011\rangle + |100\rangle + |101\rangle - |110\rangle + |111\rangle \right) \end{aligned}$$



$\sim$ EPR-pair $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

k-pairability

Notion introduced by Sergey Bravyi, Yash Sharma, Mario Szegedy, Ronald de Wolf in "Generating k EPR-pairs from an n-party resource state" (2022). Motivation : Quantum communication networks.

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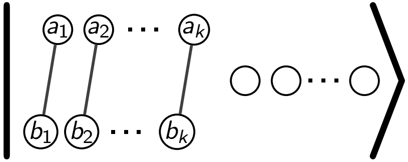
Definition (k-pairable state)

A n-qubit quantum state $|\psi\rangle$ is k-pairable if for any k disjoint pairs of qubits $\{a_1, b_1\}, \{a_2, b_2\}, \dots, \{a_k, b_k\}$ there exits an LOCC (Local Operations and Classical Communication) protocol that transforms $|\psi\rangle$ into $|\pi\rangle$.

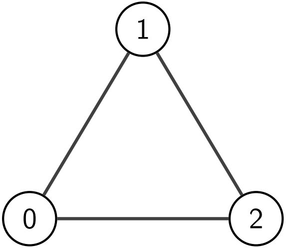
$|\pi\rangle =$



$\sim$ k EPR-pairs and $n - 2k$ isolated qubits

k-pairable states : example 1

$|K_3\rangle$ is 1-pairable.



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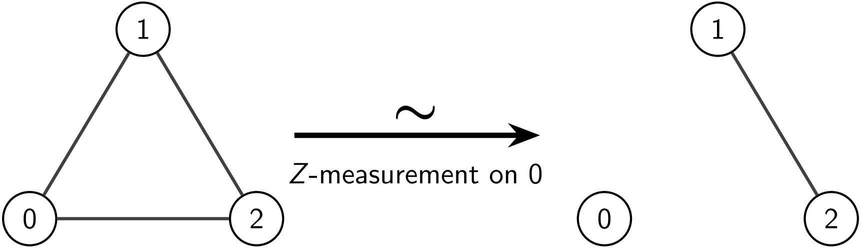

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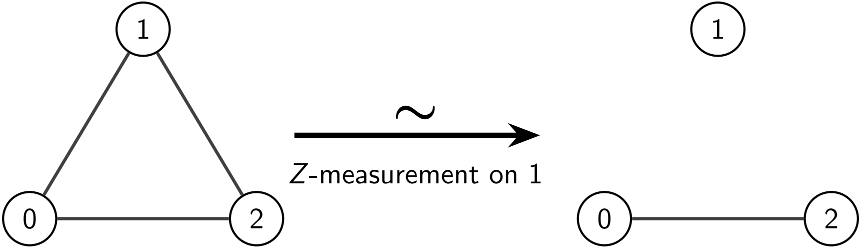

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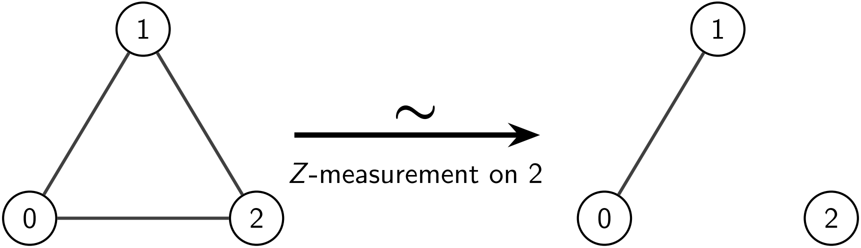
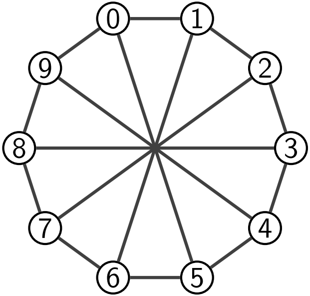


k-pairable states : example 2

$|G\rangle$ is 2-pairable.



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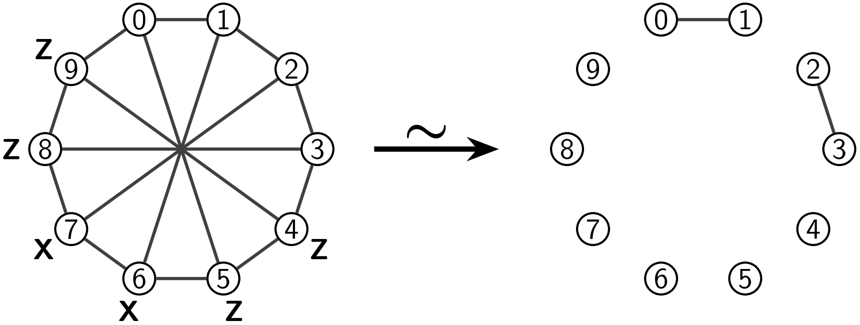

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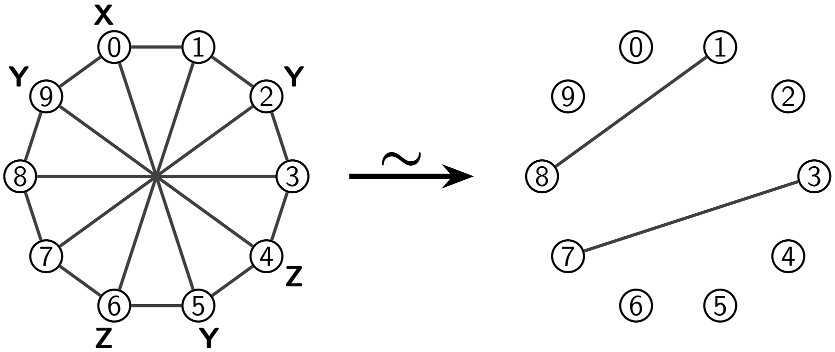


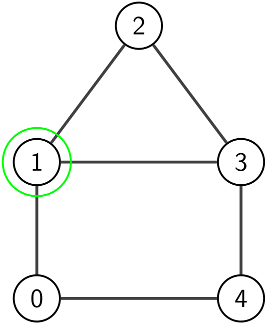
A graphical interpretation of k-pairability using vertex-minors

Sufficient condition on G for $|G\rangle$ to be k-pairable ?

Local complementation

Definition (Local complementation)

Given a graph G, a local complementation on a vertex u consists in complementing the (open) neighborhood of u in G.



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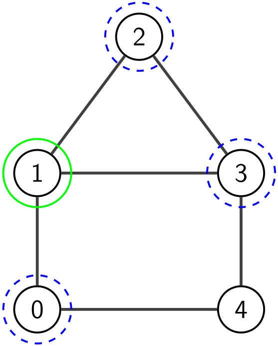

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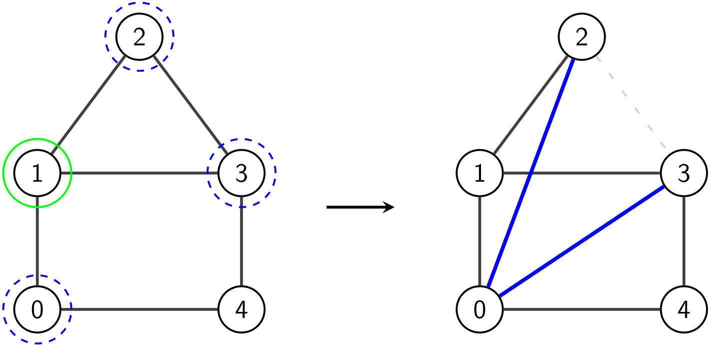

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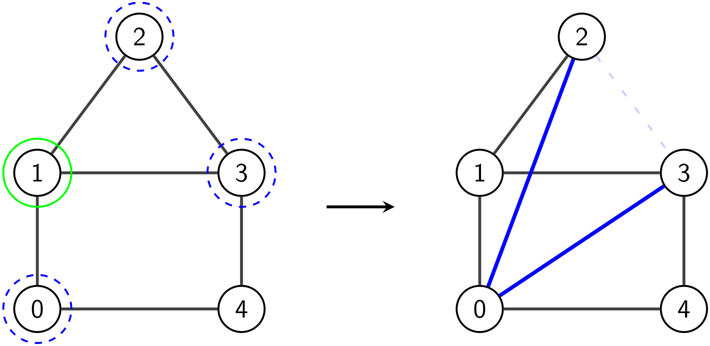
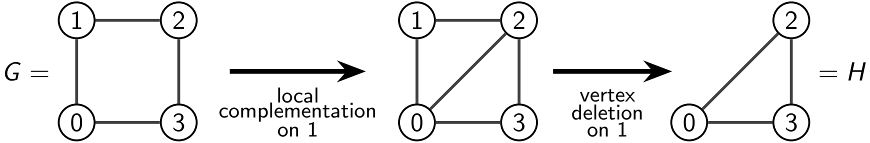


A local complementation on G can be implemented by local operations on $|G\rangle$.

Definition (Vertex-minor)

Given two graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$ such that $V_H \subseteq V_G$, H is a vertex-minor of G if H can be obtained from G by means of local complementations and vertex deletions.



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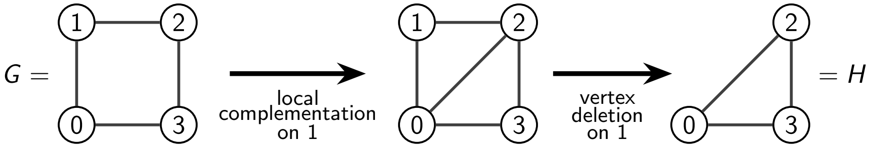
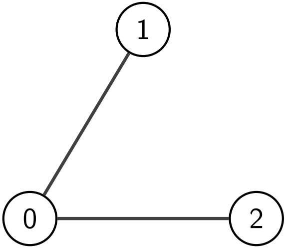


Proposition

A graph state $|G\rangle$ is k-pairable if any perfect matching on any $2k$ vertices is a vertex-minor of G.

k-pairable states : example 1

$|G\rangle$ is 1-pairable.



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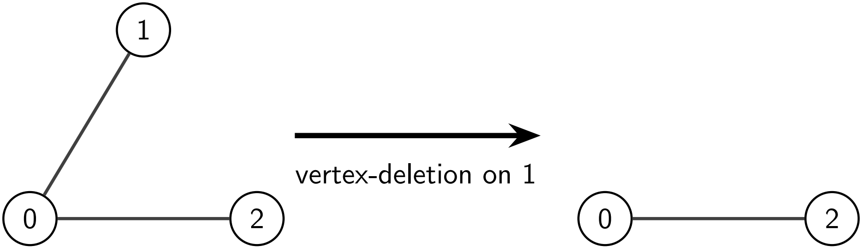

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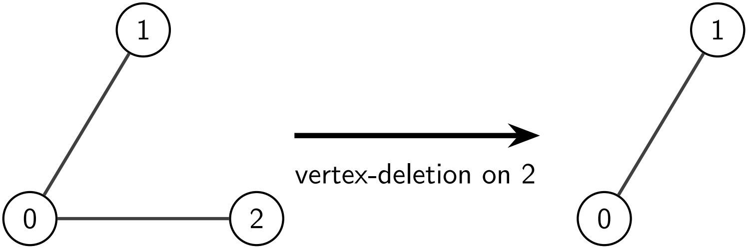

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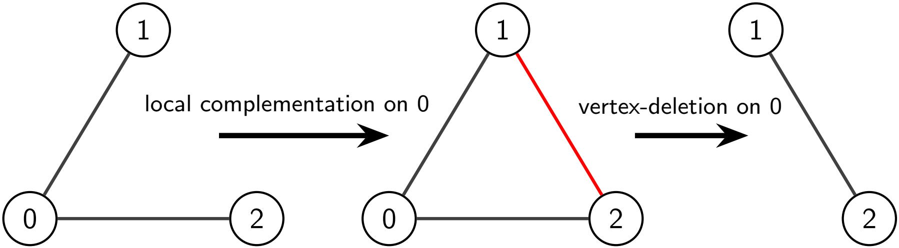
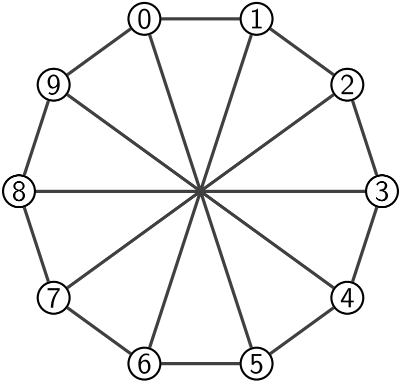


k-pairable states : example 2

$|G\rangle$ is 2-pairable.



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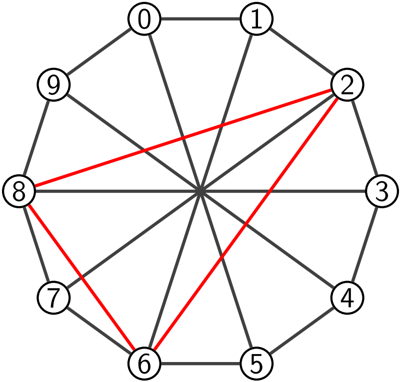

To induce the pairs $(0, 1)$ and $(2, 3)$: local complementation on 7

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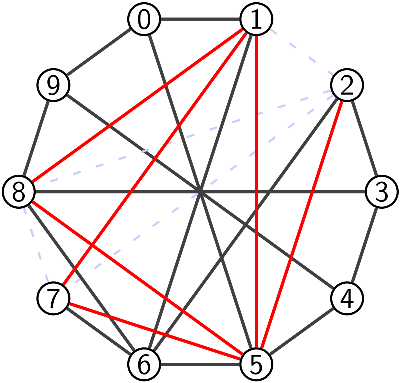

To induce the pairs $(0, 1)$ and $(2, 3)$: local complementation on 7, then local complementation on 6

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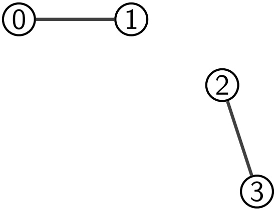

To induce the pairs $(0, 1)$ and $(2, 3)$: local complementation on 7, then local complementation on 6, then vertex-deletions on 4,5,6,7,8 and 9.

Main result : Existence of small k-pairable states

Existence of small k-pairable states

Proposition (Bravyi et al. 2022)

For any k, there exists a k-pairable state on $n = 2^{3k}$ qubits.

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Does there exists k-pairable state where $n = poly(k)$?

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For any k, there exists a k-pairable state on $n = 2^{3k}$ qubits.

Does there exists k-pairable state where $n = poly(k)$?
Yes !

Proposition

For any k, there exists a k-pairable state on $n = O(k^3 \ln^3(k))$ qubits.

k-vertex-minor universal graphs

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Recall that a sufficient condition on $|G\rangle$ to be k-pairable is that G has every perfect matching on any $2k$ vertices as its vertex-minors.

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Natural extension :

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A graph G is *k-vertex-minor universal* if any graph on any k vertices is a vertex-minor of G.

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If G is a k-vertex-minor universal graph, then one can induce any stabilizer state of on any set of k qubits in the corresponding graph state $|G\rangle$ by LOCC protocols.

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Corollary

If G is a $2k$-vertex-minor universal graph, then $|G\rangle$ is a k-pairable graph state.

Existence of small k-vertex-minors universal graphs

Proposition

For any k, there exists a k-vertex-minor universal graph of order $n = O(k^4 \ln(k))$.

Conclusion

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Results on k-pairable states and k-vertex-minor universal graphs.

Future work:

- Explicit constructions
- k-pairability $=$ $2k$-vertex-minor universality ?

Thanks

arXiv:2309.09956